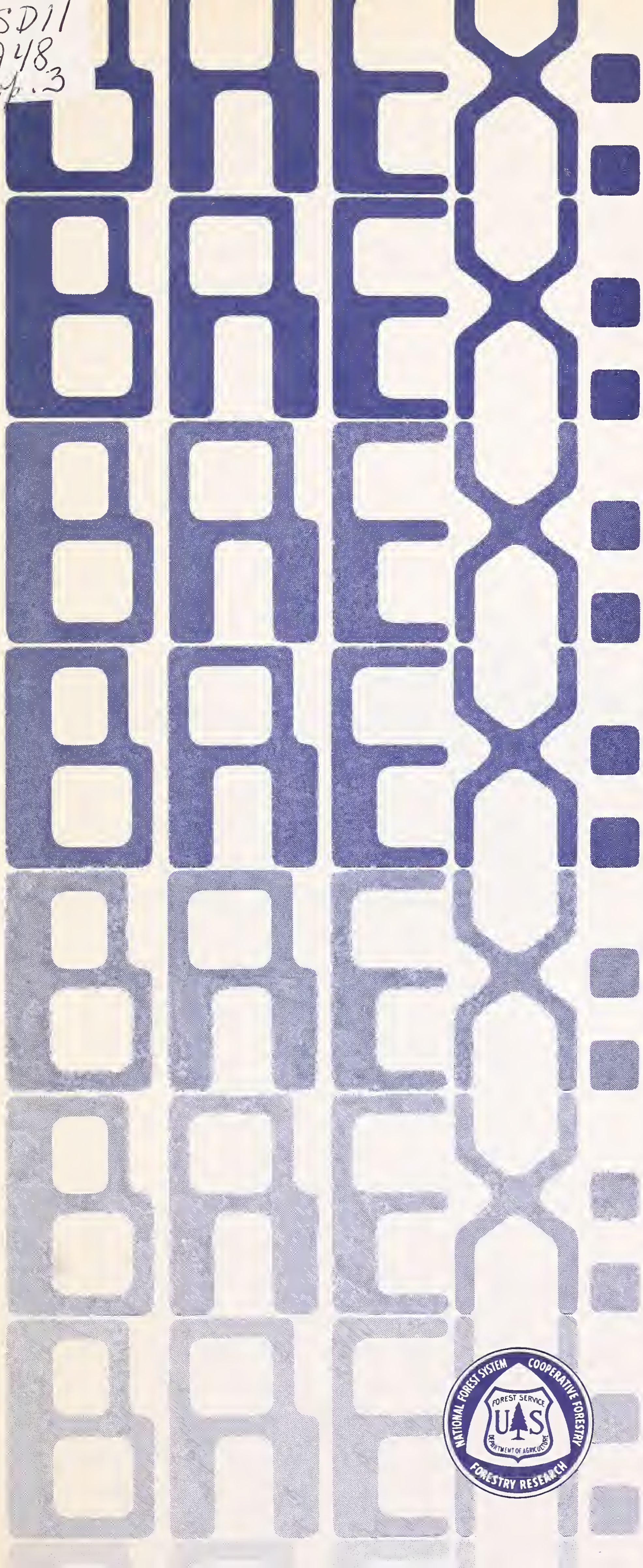

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BREX: A COMPUTER PROGRAM FOR APPLYING RIDGE REGRESSION TECHNIQUES TO MULTIPLE LINEAR REGRESSION

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RESEARCH SUMMARY

An algorithm for ridge regression (BREX) is presented which can be used in conjunction with Grosenbaugh's REX (1967), a linear regression program with combinatorial screening. The algorithm uses either REX punched matrix input, or raw data with suitable transformations, to estimate ridged (biased) regression coefficients. A modification of Marquardt's (1970) criterion is proposed for selecting the best biasing level (k value). Application of the algorithm is demonstrated on two widely-used data sets (Hoerl and Kennard, 1970b; Draper and Smith, 1966).

CONTENTS

INTRODUCTION

Multiple linear regression is one of the most commonly used statistical techniques in forestry. Regression analysis is used routinely to develop growth and yield models and to estimate individual tree and stand volumes. This process can be divided into three components: model hypothesis, screening of variables in the hypothesis, and parameter estimation for the final model.

While many criteria have been proposed for selecting the best subset of variables, screening all combinations does guarantee that the best set is actually found (Hocking 1976). Grosenbaugh (1967) developed a program, REX, that selects the independent variables by screening all combinations and picking that combination which minimizes the relative mean-squared error (relative mean-squared error is defined as the ratio of the regression mean-squared error to the variance of the dependent variable). REX then uses the least-squares criterion to estimate the regression parameters. These estimates are the best linear unbiased estimates (BLUE's) for the model, but in the presence of multicollinearity they may be too imprecise to be useful. Multicollinearity exists when at least one independent variable is highly correlated with another, or with a linear combination of other independent variables. The high degree of imprecision is due to the fact that multicollinearity between independent variables causes the correlation matrix to approach singularity and the variances of the parameter estimates to approach infinity.

When faced with multicollinearity, the analyst may be tempted to eliminate those independent variables causing the problem by consciously removing them from the model or by using a screening method, such as stepwise regression, which has a tendency to exclude such variables. However, this may destroy the usefulness of the model by eliminating relevant independent variables.

Another approach for dealing with multicollinearity is to apply ridge regression methodology. A ridged solution produces biased estimates of the model parameters, but it can also greatly reduce the variances of the parameters so that the sum of their mean-squared errors (MSE) is less than that for the least-squares solution. By combining ridge regression with a least-squares program that screens all combinations, it is possible to first select the set of variables which minimizes MSE, and then minimize any resulting multicollinearity problems by ridging the parameter estimates.

For those interested in using a model for predictive purposes, the ridged model offers two advantages over the least-squares model. First, the ridged model has a lower MSE value than the least-squares model; this increases confidence in predictions (Hoerl and Kennard 1970a). Second, the ridged coefficients can often meet theoretical or practical constraints on the model when the least-squares estimates are of wrong sign or magnitude (Hoerl and Kennard 1970b). The latter feature is particularly important if the model is to be used outside of the range of the original data set, as is often the case in simulation.

For users primarily interested in interpreting the regression coefficients, the presence of multicollinearity causes serious problems. Kmenta (1971) has shown that the least-squares estimates are highly imprecise under high multicollinearity. By using ridge regression, the precision of the regression coefficients is increased and interpretation becomes more dependable.

This report presents a computer program, BREX (biased REX), which computes ridge estimates from the moment matrix output of REX. For those who do not use REX, BREX can also be run using raw input data.

PRINCIPLES OF RIDGE REGRESSION

Ridge Regression

The multiple linear regression model is commonly written as,

$$Y = XB + e \tag{1}$$

where Y is the n vector of observations on the dependent variable, X is the nxp matrix of observations on the independent variables, B is the p vector of parameters, and e is the n vector of errors. It is assumed that X has full rank p and

$$E(e) = 0 \qquad E(ee') = \sigma^2 I \tag{2}$$

For convenience, it is also assumed that all the variables have been standardized so that the sample means are zero and the sums of squares are one. The moment matrix, $X'X$, then consists of the sample correlation coefficients (Draper and Smith 1966).

The least-squares estimates are given by

$$\hat{B} = (X'X)^{-1}X'Y \tag{3}$$

with variance-covariance matrix,

$$COV(\hat{B}_i,\hat{B}_j) = \sigma^2(X'X)^{-1} \tag{4}$$

The diagonal elements of the inverse of the correlation matrix are labeled the variance inflation factors (VIF) by Marquardt and Snee (1975) since the variances of the estimated parameters are directly proportional to these factors. If the independent variables are uncorrelated, then

$$(X'X) = I = (X'X)^{-1} \tag{5}$$

and the VIF's are all equal to one. In the presence of high correlations, however, some of the VIF's become much larger than one and the corresponding estimates are very imprecise (Marquardt 1970). Because the least-squares estimates are the BLUE's for the model, the precision can only be improved by using nonlinear or biased estimators. Nonlinear estimation tends to be computationally difficult. Among the alternatives to nonlinear estimation are ridge regression, principal components, and shrinkage estimators. Ridge regression is the most popular biased method, probably because its relationship to least-squares and its statistical properties are clearly defined.

Hoerl (1962) first suggested that adding a small positive constant k to the diagonal elements of $X'X$ would reduce the variances of the coefficients. The ridge estimates are

$$\hat{B}^* = (X'X + kI)^{-1}X'Y, \; k>0 \qquad (6)$$

with variance-covariance matrix,

$$COV(\hat{B}_i^*,\hat{B}_j^*) = \sigma^2(X'X + kI)^{-1}X'X(X'X + kI)^{-1}. \qquad (7)$$

Note that for $k = 0$ the ridge estimates are equal to the least-squares solution.

Hoerl and Kennard (1970a) have shown that although the ridge estimates are biased, there always exists a $k>0$ such that the sum of MSE's of the parameter estimates is lower than for least-squares. Hocking and others (1976) have concluded, however, that ridge regression is not necessarily superior to least-squares when the number of independent variables is less than three. Vinod (1976) and Marquardt and Snee (1975) have argued that minimal MSE is a better criterion than BLUE for judging the quality of regression estimators. An estimator that is close to the true value with high probability, though biased, is thought better than an unbiased estimator which has a low probability of being near the true value.

Selecting a "k" Value

Although ridging can always reduce MSE for nonorthogonal regressors, no closed-form solution exists for the k value that minimizes the summed MSE. Hoerl and Kennard (1970b) proposed examining the ridged parameter estimates, $\hat{B}^*$, over a range of k values (the ridge trace) and selecting one where $\hat{B}^*$ stabilizes. Since we are interested mainly in the variances of these estimates, this approach is indirect as well as very subjective. Stability is a matter of degree since, as k goes to infinity, $\hat{B}^*$ approaches zero.

The situation is simplified if we transform the variables to their principal components (linear combinations of the variables which are mutually orthogonal) to obtain a generalization of ridge regression. Following Hilt and Seequist (1977), there always exists an orthogonal transformation matrix P such that,

$$P'X'XP = D \text{ and } P'P = I. \qquad (8)$$

The columns of P are the eigenvectors of $X'X$, and D is the diagonal matrix of corresponding eigenvalues. The least-squares regression coefficients on the principal components are given by

$$\hat{A} = D^{-1} P'X'Y \qquad (9)$$

and the generalized ridge solution by

$$\hat{A}^* = (D + K)^{-1}P'X'Y \qquad (10)$$

where K is a diagonal matrix with nonnegative elements k_i. The MSE of each element $\hat{a}_i$ of $\hat{A}^*$ is minimized by setting $k_i = \sigma^2/a_i^2$ (Hoerl and Kennard 1970a). These optimal k_i's depend on the unknown parameters σ^2 and a_i which can be replaced with the consistent estimates, $\hat{\sigma}^2$ and $(\hat{a}_i)^2$. However, this substitution destroys the minimum MSE property (Vinod 1976). Since these least-squares estimates are generally poor due to ill conditioning, Hoerl and Kennard (1976) suggest that they only be used as a starting point to estimate initial values of the k_i's. These k_i's are then used to find a new $\hat{A}^*$ and the iteration continues until convergence of the k_i's to stationary values occurs. Hemmerle (1975) presents a closed-form solution to this iterative process, but finds that it often leads to an unacceptable increase in the error sum of squares (ESS). He proposes allocating the allowable inflation of ESS proportionally to each principal component. This method brings those a_i's previously set to zero back into the solution, though they obviously contribute the most to variance inflation (Hocking and others 1976). Until a more satisfactory procedure for constraining the increase in ESS is found, generalized ridge regression lacks practicality.

With ordinary ridge regression (k_i = constant), one could average the generalized k_i's to find a single k value, the harmonic mean being suggested (Hoerl and others 1975), to minimize the effects of small a_i's which have little predictive value. This approach leads to the estimator,

$$\hat{k} = p\hat{\sigma}^2/\hat{A}'\hat{A} = p\hat{\sigma}^2/\hat{B}'\hat{B} \qquad (11)$$

where p is the number of independent variables (the denominator is the length of the parameter vector which is unchanged by the switch to principal components). The same estimator arises by minimizing the summed MSE of the parameter estimates for the special case where $X'X = I$. Again, the least-squares estimates are used as a starting point and then the iteration is continued on $(\hat{B}^*)'\hat{B}^*$ until the k value converges.

This procedure was included as an option in an earlier version of BREX. We found that the initial least-squares estimates often produced k greater than one and convergence to infinity occurred. Mallows' (1973) estimator,

$$\hat{k} = p\hat{\sigma}^2/(\hat{B}'\hat{B} - p\hat{\sigma}^2) \qquad (12)$$

was also considered since it is unbiased, but, as it is always larger than that of Hoerl and others (1975), the same problem would occur.

The most promising method for selecting a k value was that of Marquardt (1970). He sought a k value such that the maximum VIF was between 10 and 1, and closer to 1. Obviously, VIF's less than 1 are undesirable because 1 is the lower limit attained by a perfectly orthogonal system. BREX calculates both minimum and maximum VIF's, the former being a secondary criterion which preferably would be not much less than 1. Defining a desirable range such as this allows consideration of the increase in the ESS as well.

A further advantage of Marquardt's criterion is that k is nonstochastic, depending only on fixed X (Obenchain 1975). This property is required if the equations derived by Hoerl and Kennard (1970a) for $E(\hat{B}^*)$ and $COV(\hat{B}_i^*, \hat{B}_j^*)$ are to be valid. The estimators previously discussed are stochastic, depending on Y, and therefore, technically, their moments are unknown.

PROGRAM OPERATION

Program Availability

BREX is written in FORTRAN IV and is operational on a CDC 6400 and a CDC CYBER 73. A copy of the program can be obtained by writing:

PROGRAM BREX
Renewable Resources Evaluation Research Work Unit
507 - 25th Street
Ogden, Utah 84401

Features of BREX

BREX was specifically designed to be used in conjunction with Grosenbaugh's REX. For this reason, it uses as input the moment matrix punched by REX. To allow access by other users, subroutines MTRX and TRNX were added for the conversion of raw data to REX matrix form.

Control card input is assumed to be from cards, but the data matrix or observations may be read from tape. If raw data are used, punched output identical to that of REX can be requested for later reuse. For a large number of observations, efficiency is greatly increased by direct use of the moment matrix.

For each problem to be solved, the user can select ridge output for up to 10 specified k values or accept the default selection of 20 values over the range 0.005 to 1.0. Like REX, the user can specify the use of the moment matrix from the previous problem for any problem except the first of a run. In this way, many possible models can be checked on a single run while only reading in the data once.

The first page of the output gives the problem title and a statement of parameters. For raw data, the first two and the last transformed observation vectors are displayed as a check against errors in TRNX or an incorrect number of observations.

The second page presents the least-squares solution with the standardized coefficients, and the normal coefficients and their variances. Also displayed is the estimated MSE, ESS, minimum and maximum VIF's and $\overline{R}^2$. Succeeding pages show the ridge solution for each k value, giving statistics similar to those for the least-squares solution.

Control Cards for the Use of BREX

Field Function	Column	Name	Format
Card 1			
Problem identification	5-76	NAME	18A4
Label for punched output (from raw data only)	77-80	NAME	A4
Card 2			
Total number of variables in REX input matrix or number of variables after transformation of raw data	1-5	NNT	I5
Number of independent variables for this problem	6-10	NGX	I5
Number of *k* values to be read in (0 = default selection)	11-15	NNK	I5
Use data from previous problem? (Yes = 1, No = 0)	16-20	IPD	I5
Should regression be performed without intercept?[1] (Yes = 1, No = 0)	21-25	IMO	I5
Are input data from tape?[2] (Yes = tape number, No = 0)	26-30	ICT	I5
Are the observations to be weighted?[1] (Yes = 1, No = 0)	31-35	IW	I5
Number of observations for raw data input (= 0 for REX matrix input)	36-45	NOB	I10
Number of variables (excluding weight) for each raw observation vector (= 0 for REX matrix input)	46-50	NVAR	I5
Tape number for punched output[2] (= 0 for REX matrix input)	51-55	IPO	I5

(con.)

[1]Unlike REX, these options have no effect on REX matrix input except to provide correct labeling on the first page of output. Also, they cannot be used to change these specifications for a problem using data from a previous one. It is important to repeat the same specifications for previous data problems so that the output will be correctly labeled.

[2]The first statement of the main program specifies tape 7 for the punch and tape 10 for data input. If other unit numbers must be used, simply change the numbers in this statement.

Control Cards for the Use of BREX (Cont.)

Field Function	Column	Name	Format
Card 3			
Punch X's and a Y in the columns whose ordinals are the subscripts of the independent variables and the dependent variable, respectively	1-50	NX	50A1
Card 4			
Up to 10 k values to be used, in order of ascending magnitude (blank for default selection)	1-80	XK	10F8.4
Cards 5 and 6			
Variable format for raw data input (blank for REX matrix input)	1-80 /1-80	FMT	20A4 /20A4

These six cards are followed by the input data if they are from cards. This sequence of cards (1-6 plus data cards, if any) is repeated for each problem in the run. After the last problem set, a card punched "DONE" in columns 1-4 signals the end of the run.

Subroutine TRNX

This is a user-supplied subroutine that generates regression variables from raw data using FORTRAN statements. The input variables, $D(I)$, I = 1, NVAR, are read according to the variable format on control cards 5 and 6 of the problem set. The output variables, $X(I)$, I = 1, NNT, and the weight W, then are used to form the $X'X$ matrix. TRNX is called once for each observation to assign values for all the independent and dependent variables to be used for all the problems of a run. For each problem set, control card 3 specifies the dependent variable and regressor variables in the particular model.

The cards labeled TRNX 10 through TRNX 110 are always required. In the example below, the raw variables are used untransformed with a constant weight of one.

```
      SUBROUTINE TRNX                                                    TRNX  10
C                                                                        TRNX  20
C SUBROUTINE TRNX ALLOWS NVAR RAW INPUT VARIABLES D(I) TO BE             TRNX  30
C TRANSFORMED INTO (NGX+1) REGRESSION VARIABLES USING FORTRAN.           TRNX  40
C IF WEIGHTED REGRESSION, W MUST BE ASSIGNED A VALUE                     TRNX  50
C                                                                        TRNX  60
      COMMON/RMT/X                                                       TRNX  70
      COMMON/MT/D,W                                                      TRNX  80
      DIMENSION  D(50), X(50)                                            TRNX  90
      DO 1  I = 1, NVAR
      X(I) = D(I)
    1 CONTINUE
      W = 1.0
      RETURN                                                             TRNX 100
      END                                                                TRNX 110
```

APPLICATIONS OF BREX

To demonstrate the use of BREX, we chose two data sets, the Gorman-Toman 10 variable set (Hoerl and Kennard 1970b, p. 71) and the Hald 4 variable set (Draper and Smith 1966, p. 366). The former has been used in a number of papers on ridge regression (Mallows 1973, Marquardt and Snee 1975, Obenchain 1975, and Vinod 1976), and the latter is an excellent example of severe multicollinearity. The computer output generated by these example data sets is found in appendix B.

Analysis of the Gorman-Toman Data

Since this data set was presented in the form of a correlation matrix, we used it directly as a moment matrix punched in REX format. The normal coefficients are identical to the standardized coefficients because a correlation matrix was used as input. The regression was specified as unweighted and through the mean with matrix input from tape 10 (see sample output, p. 13).

For the least-squares solution (see sample output, p. 13), the maximum VIF was just under 10, so one would expect a fairly low k value. Marquardt's criterion is satisfied for all k values less than 0.2 (see p. 14-17) but we would select k = 0.04 as this is the maximum k value in that range for which the minimum VIF is still greater than one. This ridged solution has reduced the maximum VIF by 58 percent with only a 9 percent increase in the RSS relative to least-squares. Hoerl and Kennard (1970b) selected k between 0.2 and 0.3 using the ridge trace. We would conclude that their approach leads to overestimation of the amount of bias necessary to stabilize the system. Note that the default selection of k values was truncated at k = 0.2 as the maximum VIF fell under 1.0.

Analysis of the Hald Data

In this example, the raw data (13 observations) were read from cards and punched moment matrix output was requested (see sample output, p. 19). The data were weighted by 1.0, to show the change in labeling, and were used untransformed by TRNX.

This data set was severely multicollinear as indicated by a maximum VIF of 282 for the least-squares solution (see p. 20). Using our modified Marquardt criterion, we selected k = 0.03 which reduced the maximum VIF by over 99 percent with a 30 percent increase in RSS. $\overline{R}^2$ decreased less than 2 percent. The determinant of $X'X$ has increased more than 10 times indicating the gain in stability.

The second problem set (see p. 24-25) in this run used only the variables 1 and 2 for prediction. The data matrix from the first problem is used with three selected k values, 0.11, 0.13, and 0.15. The least-squares solution is almost orthogonal with both minimum and maximum VIF's barely larger than one. Ridging is obviously unnecessary, and those solutions are truncated after the first two k values.

Note that indicators for weighting and for an intercept are repeated (see p. 24) so that the problem is correctly labeled. However, the indicators have no effect on the solution.

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APPENDIX A

Programing Notes

The main program, BREX, interprets the control cards for each problem set. If raw data are input, subroutine MTRX is called to read them using subroutine TRNX to generate variables for the regressions. MTRX also produces punched output of the moment matrix in vector S, identical to that of REX, if requested. The elements of S are corrected for the mean if IMO = 0 and the weights are adjusted to sum to the number of observations BREX then creates the correlation matrix for the model specified on control card 3 from the vector S. The standardized least-squares solution is found by inverting the correlation matrix using subroutine NVRT. Gauss-Jordan elimination with column pivoting to maximize the pivot elements is the technique used. The inversion is performed in double precision on the scaled (correlation) form of the matrix to achieve a minimum of rounding error (Deegan 1976). As a check for numerical singularity, the determinant of $X'X$ is output.

The normal regression coefficients and their variances are obtained from the standardized solution (Draper and Smith 1966). The estimates of the MSE, ESS, and adjusted R^2 ($\bar{R}^2$) are all based on the normalized solution.

The ridged solutions are calculated in a similar manner, first adding k to the diagonal of the correlation matrix. Since the residuals are not available from matrix input, the RSS is calculated by

$$\text{RSS}(k) = Y'Y - (B^*)'X'Y - k(B^*)'B^* \qquad (13)$$

as given by Hoerl and Kennard (1970a).

APPENDIX B

Card Deck Setup and Output Generated by Example Data Sets
Card Deck Setup for Gorman-Toman Data (CDC's Scope or NOS / BE Operating Systems)

CM60000 on the job card indicates that, at most, 60,000 (octal) words of memory are required to run this program. The zeros on BREX control card 2 need not be punched, as a blank is read as a zero.

```
JOBNAME,CM60000.
ATTACH(TAPE10,GTDATA,ID=MITCH)
REWIND(TAPE10)
FORTRAN.
LGO.
*EOR  (end of record card)
  (followed by the FORTRAN source deck BREX and subroutines MTRX and NVRT)
SUBROUTINE TRNX
COMMON/RMT/X
COMMON/MT/D,W
DIMENSION  D(50), X(50)
RETURN
END
*EOR
    GORMAN TOMAN  TEN FACTOR PROBLEM
   11   10    0    0    0   10    0           0    0    0
XXXXXXXXXXY
blank card
blank card
blank card
DONE
*EOF  (end of file card)
```

```
GORMAN TOMAN  TEN FACTOR PROBLEM

UNWEIGHTED REGRESSION THRU THE MEAN    USING REX DATA

NO. OF VARIABLES READ IN = 11 ON CARDS


VARIABLES IN REGRESSION

                1    1    2    2    3    3    4    4    5
           5    0    5    0    5    0    5    0    5    0
====XXXXXXXXXXY


DEFAULT SELECTION OF K VALUES REQUESTED



LEAST-SQUARES SOLUTION

MSE      =  4.1308E-03          DETERMINANT =  3.9954E-03

MAX VIF  =  9.3315E+00          RESIDUAL SS =  1.0327E-01

MIN VIF  =  1.1589E+00          RBARSQ      =  8.5542E-01
```

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-1.8504850E-01	2.1227045E-02	-1.8504850E-01
2	-2.2144353E-01	6.9481955E-03	-2.2144353E-01
3	-3.5946180E-01	9.1378250E-03	-3.5946180E-01
4	-1.0541815E-01	4.7873375E-03	-1.0541815E-01
5	-4.6891860E-01	2.5177717E-02	-4.6891860E-01
6	8.1316918E-01	3.8546446E-02	8.1316918E-01
7	2.8474318E-01	1.0008915E-02	2.8474318E-01
8	3.8337884E-01	8.5157305E-03	3.8337884E-01
9	9.2058062E-02	5.4828320E-03	9.2058062E-02
10	9.4442659E-02	9.8932603E-03	9.4442659E-02
U	.0		.0

GORMAN TOMAN TEN FACTOR PROBLEM

RIDGED (BIASED) SOLUTIONS

K = 5.0000E-03   DETERMINANT = 4.7150E-03

MAX VIF = 8.1445E+00   RESIDUAL SS = 1.0353E-01

MIN VIF = 1.1375E+00   RBARSQ = 8.5505E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-2.0125588E-01	1.9336351E-02	-2.0125588E-01
2	-2.1398027E-01	6.6957033E-03	-2.1398027E-01
3	-3.5499681E-01	8.8507526E-03	-3.5499681E-01
4	-1.0223073E-01	4.6986361E-03	-1.0223073E-01
5	-4.3718196E-01	2.2880809E-02	-4.3718196E-01
6	7.7006222E-01	3.3643492E-02	7.7006222E-01
7	2.6901578E-01	9.4527598E-03	2.6901578E-01
8	3.7743756E-01	8.2087548E-03	3.7743756E-01
9	9.6559720E-02	5.3213082E-03	9.6559720E-02
10	9.8200681E-02	9.5458876E-03	9.8200681E-02
U	.0		.0

K = 1.0000E-02   DETERMINANT = 5.5280E-03

MAX VIF = 7.1765E+00   RESIDUAL SS = 1.0421E-01

MIN VIF = 1.1173E+00   RBARSQ = 8.5411E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-2.1489941E-01	1.7732604E-02	-2.1489941E-01
2	-2.0732151E-01	6.4677241E-03	-2.0732151E-01
3	-3.5069034E-01	8.5794495E-03	-3.5069034E-01
4	-9.9365396E-02	4.6155381E-03	-9.9365396E-02
5	-4.0902430E-01	2.0930306E-02	-4.0902430E-01
6	7.3262822E-01	2.9644613E-02	7.3262822E-01
7	2.5493465E-01	8.9675271E-03	2.5493465E-01
8	3.7193814E-01	7.9254569E-03	3.7193814E-01
9	1.0045223E-01	5.1751752E-03	1.0045223E-01
10	1.0146399E-01	9.2213807E-03	1.0146399E-01
U	.0		.0

RIDGED (BIASED) SOLUTIONS

K = 2.0000E-02   DETERMINANT = 7.4706E-03

MAX VIF = 5.7072E+00   RESIDUAL SS = 1.0630E-01

MIN VIF = 1.0804E+00   RBARSQ = 8.5118E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-2.3635977E-01	1.5161208E-02	-2.3635977E-01
2	-1.9589621E-01	6.0692427E-03	-1.9589621E-01
3	-3.4253233E-01	8.0793245E-03	-3.4253233E-01
4	-9.4395176E-02	4.4630568E-03	-9.4395176E-02
5	-3.6114761E-01	1.7798997E-02	-3.6114761E-01
6	6.7086778E-01	2.3575393E-02	6.7086778E-01
7	2.3067894E-01	8.1567040E-03	2.3067894E-01
8	3.6198113E-01	7.4181487E-03	3.6198113E-01
9	1.0679811E-01	4.9193724E-03	1.0679811E-01
10	1.0682439E-01	8.6311054E-03	1.0682439E-01
U	.0		.0

K = 3.0000E-02   DETERMINANT = 9.9027E-03

MAX VIF = 4.6596E+00   RESIDUAL SS = 1.0890E-01

MIN VIF = 1.0471E+00   RBARSQ = 8.4754E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-2.5218034E-01	1.3192863E-02	-2.5218034E-01
2	-1.8638641E-01	5.7293226E-03	-1.8638641E-01
3	-3.3494131E-01	7.6289630E-03	-3.3494131E-01
4	-9.0199323E-02	4.3252462E-03	-9.0199323E-02
5	-3.2183073E-01	1.5399404E-02	-3.2183073E-01
6	6.2206816E-01	1.9248003E-02	6.2206816E-01
7	2.1042125E-01	7.5006885E-03	2.1042125E-01
8	3.5308820E-01	6.9753570E-03	3.5308820E-01
9	1.1168298E-01	4.7009305E-03	1.1168298E-01
10	1.1100280E-01	8.1067405E-03	1.1100280E-01
U	.0		.0

GORMAN TOMAN TEN FACTOR PROBLEM

RIDGED (BIASED) SOLUTIONS

K = 4.0000E-02 DETERMINANT = 1.2915E-02

MAX VIF = 3.8857E+00 RESIDUAL SS = 1.1172E-01

MIN VIF = 1.0165E+00 RBARSQ = 8.4360E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-2.6406036E-01	1.1640049E-02	-2.6406036E-01
2	-1.7829442E-01	5.4335251E-03	-1.7829442E-01
3	-3.2786825E-01	7.2214413E-03	-3.2786825E-01
4	-8.6581606E-02	4.1990928E-03	-8.6581606E-02
5	-2.8885650E-01	1.3505543E-02	-2.8885650E-01
6	5.8257458E-01	1.6050971E-02	5.8257458E-01
7	1.9316162E-01	6.9548119E-03	1.9316162E-01
8	3.4499789E-01	6.5842768E-03	3.4499789E-01
9	1.1548683E-01	4.5106075E-03	1.1548683E-01
10	1.1430727E-01	7.6370651E-03	1.1430727E-01
U	.0		.0

K = 5.0000E-02 DETERMINANT = 1.6610E-02

MAX VIF = 3.2971E+00 RESIDUAL SS = 1.1459E-01

MIN VIF = 9.8831E-01 RBARSQ = 8.3958E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-2.7310025E-01	1.0385666E-02	-2.7310025E-01
2	-1.7128685E-01	5.1723008E-03	-1.7128685E-01
3	-3.2126607E-01	6.8510528E-03	-3.2126607E-01
4	-8.3410475E-02	4.0825087E-03	-8.3410475E-02
5	-2.6073167E-01	1.1975855E-02	-2.6073167E-01
6	5.4997853E-01	1.3619818E-02	5.4997853E-01
7	1.7822368E-01	6.4908692E-03	1.7822368E-01
8	3.3753830E-01	6.2355812E-03	3.3753830E-01
9	1.1846770E-01	4.3421565E-03	1.1846770E-01
10	1.1694606E-01	7.2136166E-03	1.1694606E-01
U	.0		.0

RIDGED (BIASED) SOLUTIONS

K = 6.0000E-02 DETERMINANT = 2.1105E-02

MAX VIF = 2.8387E+00 RESIDUAL SS = 1.1745E-01

MIN VIF = 9.6203E-01 RBARSQ = 8.3557E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-2.8004042E-01	9.3528089E-03	-2.8004042E-01
2	-1.6513114E-01	4.9389754E-03	-1.6513114E-01
3	-3.1509129E-01	6.5130527E-03	-3.1509129E-01
4	-8.0593967E-02	3.9739733E-03	-8.0593967E-02
5	-2.3641005E-01	1.0717087E-02	-2.3641005E-01
6	5.2262982E-01	1.1726256E-02	5.2262982E-01
7	1.6513084E-01	6.0900413E-03	1.6513084E-01
8	3.3059118E-01	5.9222325E-03	3.3059118E-01
9	1.2080767E-01	4.1911641E-03	1.2080767E-01
10	1.1906577E-01	6.8297742E-03	1.1906577E-01
U	.0		.0

K = 7.0000E-02 DETERMINANT = 2.6533E-02

MAX VIF = 2.4744E+00 RESIDUAL SS = 1.2025E-01

MIN VIF = 9.3743E-01 RBARSQ = 8.3165E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-2.8539501E-01	8.4888576E-03	-2.8539501E-01
2	-1.5965951E-01	4.7286764E-03	-1.5965951E-01
3	-3.0930449E-01	6.2034638E-03	-3.0930449E-01
4	-7.8065519E-02	3.8723358E-03	-7.8065519E-02
5	-2.1513455E-01	9.6652580E-03	-2.1513455E-01
6	4.9936096E-01	1.0221342E-02	4.9936096E-01
7	1.5353590E-01	5.7391955E-03	1.5353590E-01
8	3.2407194E-01	5.6387753E-03	3.2407194E-01
9	1.2263927E-01	4.0544030E-03	1.2263927E-01
10	1.2077296E-01	6.4802205E-03	1.2077296E-01
U	.0		.0

GORMAN TOMAN TEN FACTOR PROBLEM

RIDGED (BIASED) SOLUTIONS

K = 8.0000E-02 DETERMINANT = 3.3041E-02

MAX VIF = 2.1798E+00 RESIDUAL SS = 1.2299E-01

MIN VIF = 9.1428E-01 RBARSQ = 8.2782E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-2.8953059E-01	7.7565789E-03	-2.8953059E-01
2	-1.5474751E-01	4.5377214E-03	-1.5474751E-01
3	-3.0387032E-01	5.9189283E-03	-3.0387032E-01
4	-7.5775520E-02	3.7766988E-03	-7.5775520E-02
5	-1.9634223E-01	8.7749804E-03	-1.9634223E-01
6	4.7932343E-01	9.0045110E-03	4.7932343E-01
7	1.4317867E-01	5.4288226E-03	1.4317867E-01
8	3.1791803E-01	5.3808899E-03	3.1791803E-01
9	1.2406154E-01	3.9294465E-03	1.2406154E-01
10	1.2214723E-01	6.1606018E-03	1.2214723E-01
U	.0		.0

K = 9.0000E-02 DETERMINANT = 4.0796E-02

MAX VIF = 1.9399E+00 RESIDUAL SS = 1.2565E-01

MIN VIF = 8.9240E-01 RBARSQ = 8.2410E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-2.9271455E-01	7.1288758E-03	-2.9271455E-01
2	-1.5030057E-01	4.3632497E-03	-1.5030057E-01
3	-2.9875714E-01	5.6565924E-03	-2.9875714E-01
4	-7.3686063E-02	3.6863454E-03	-7.3686063E-02
5	-1.7960487E-01	8.0131456E-03	-1.7960487E-01
6	4.6188623E-01	8.0058522E-03	4.6188623E-01
7	1.3385954E-01	5.1518234E-03	1.3385954E-01
8	3.1208175E-01	5.1450959E-03	3.1208175E-01
9	1.2515006E-01	3.8144284E-03	1.2515006E-01
10	1.2324938E-01	5.8673001E-03	1.2324938E-01
U	.0		.0

RIDGED (BIASED) SOLUTIONS

K = 1.0000E-01 DETERMINANT = 4.9984E-02

MAX VIF = 1.7805E+00 RESIDUAL SS = 1.2823E-01

MIN VIF = 8.7167E-01 RBARSQ = 8.2048E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-2.9514574E-01	6.5855541E-03	-2.9514574E-01
2	-1.4624535E-01	4.2029877E-03	-1.4624535E-01
3	-2.9393673E-01	5.4140159E-03	-2.9393673E-01
4	-7.1767557E-02	3.6006919E-03	-7.1767557E-02
5	-1.6459023E-01	7.3550291E-03	-1.6459023E-01
6	4.4657076E-01	7.1755274E-03	4.4657076E-01
7	1.2542218E-01	4.9027609E-03	1.2542218E-01
8	3.0652582E-01	4.9285479E-03	3.0652582E-01
9	1.2596360E-01	3.7078856E-03	1.2596360E-01
10	1.2412672E-01	5.5972728E-03	1.2412672E-01
U	.0		.0

K = 1.2000E-01 DETERMINANT = 7.3516E-02

MAX VIF = 1.5200E+00 RESIDUAL SS = 1.3316E-01

MIN VIF = 8.3316E-01 RBARSQ = 8.1357E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-2.9831672E-01	5.6940929E-03	-2.9831672E-01
2	-1.3909004E-01	3.9180606E-03	-1.3909004E-01
3	-2.8507613E-01	4.9800304E-03	-2.8507613E-01
4	-6.8353783E-02	3.4416300E-03	-6.8353783E-02
5	-1.3873261E-01	6.2788355E-03	-1.3873261E-01
6	4.2090765E-01	5.8839502E-03	4.2090765E-01
7	1.1071842E-01	4.4722940E-03	1.1071842E-01
8	2.9614105E-01	4.5441562E-03	2.9614105E-01
9	1.2694213E-01	3.5157858E-03	1.2694213E-01
10	1.2534929E-01	5.1170799E-03	1.2534929E-01
U	.0		.0

GORMAN TOMAN TEN FACTOR PROBLEM

RIDGED (BIASED) SOLUTIONS

K = 1.5000E-01 DETERMINANT = 1.2557E-01

MAX VIF = 1.2317E+00 RESIDUAL SS = 1.4008E-01

MIN VIF = 7.8155E-01 RBARSQ = 8.0389E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-3.0023079E-01	4.7010419E-03	-3.0023079E-01
2	-1.3016786E-01	3.5604640E-03	-1.3016786E-01
3	-2.7343191E-01	4.4331081E-03	-2.7343191E-01
4	-6.4052924E-02	3.2284178E-03	-6.4052924E-02
5	-1.0776736E-01	5.0877874E-03	-1.0776736E-01
6	3.9130329E-01	4.5529624E-03	3.9130329E-01
7	9.2827477E-02	3.9537111E-03	9.2827477E-02
8	2.8207215E-01	4.0640363E-03	2.8207215E-01
9	1.2724922E-01	3.2683124E-03	1.2724922E-01
10	1.2622555E-01	4.5175468E-03	1.2622555E-01
U	.0		.0

K = 2.0000E-01 DETERMINANT = 2.7982E-01

MAX VIF = 9.1545E-01 RESIDUAL SS = 1.5056E-01

MIN VIF = 7.0807E-01 RBARSQ = 7.8922E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	-2.9925647E-01	3.5987492E-03	-2.9925647E-01
2	-1.1854596E-01	3.0950444E-03	-1.1854596E-01
3	-2.5739977E-01	3.7241707E-03	-2.5739977E-01
4	-5.8403030E-02	2.9248868E-03	-5.8403030E-02
5	-6.9909196E-02	3.7815227E-03	-6.9909196E-02
6	3.5653716E-01	3.2159147E-03	3.5653716E-01
7	7.0583670E-02	3.3139143E-03	7.0583670E-02
8	2.6173571E-01	3.4460609E-03	2.6173571E-01
9	1.2590888E-01	2.9315614E-03	1.2590888E-01
10	1.2614206E-01	3.7509075E-03	1.2614206E-01
U	.0		.0

Card Deck Setup for Hald Data Using a Binary Version of BREX (CDC's or NOS / BE Operating Systems)

This example uses a binary (compiled) version of BREX and subroutines NVRT and MTRX, which reside on the permanent file BINARYBREX. A source deck of TRNX is used because it must be adjusted to suit a given run.

```
JOBNAME, CM60000.
ATTACH(BREX, BINARYBREX,ID=MITCH)
REWIND(BREX)
FORTRAN.
LOAD(LGO)
BREX.
*EOR  (end of record card)
SUBROUTINE TRNX
COMMON/RMT/X
COMMON/MT/D,W
DIMENSION  D(50), X(50)
DO 1 J = 1,5
1 X(J) = D(J)
W = 1.0
RETURN
END
*EOR
    DATA FROM HALD GIVEN BY DRAPER AND SMITH (1966) P.395
    5    4    0    0    0    0    1         13    5    7
XXXXY
blank card
(4F3.0,F6.1)
blank card
  7 26  6 60  78.5
  1 29 15 52  74.3
  .  .  .  .    .
  .  .  .  .    .
  .  .  .  .    .
 10 68  8 12 109.4
    DATA FROM HALD GIVEN BY DRAPER AND SMITH (1966) P.375
    5    2    3    1    0    0    1          0    0    0
XX  Y
  0.11     0.13     0.15
blank card
blank card
DONE
*EOF   (end of file card)
```

```
DATA FROM HALD GIVEN BY DRAPER AND SMITH (1966) P.395

WEIGHTED REGRESSION THRU THE MEAN    USING RAW DATA

NO. OF VARIABLES READ IN =  5 ON CARDS


VARIABLES IN REGRESSION

            1    1    2    2    3    3    4    4    5
       5    0    5    0    5    0    5    0    5    0
====XXXXYW


DEFAULT SELECTION OF K VALUES REQUESTED


DATA INPUT FORMAT
(4F3.0,F6.1)


LISTING OF FIRST TWO OBSERVATION VECTORS (AFTER TRANSFORMATIONS, IF ANY)


7.0000000E+00 2.6000000E+01 6.0000000E+00 6.0000000E+01 7.8500000E+01 1.0000000E+00
1.0000000E+00 2.9000000E+01 1.5000000E+01 5.2000000E+01 7.4300000E+01 1.0000000E+00


LAST OBSERVATION VECTOR----IF ABSENT, INCORRECT NO. OF OBS OR FORMAT


1.0000000E+01 6.8000000E+01 8.0000000E+00 1.2000000E+01 1.0940000E+02 1.0000000E+00
```

DATA FROM HALD GIVEN BY DRAPER AND SMITH (1966) P.395

LEAST-SQUARES SOLUTION

MSE	= 5.9830E+00	DETERMINANT	= 1.0677E-03
MAX VIF	= 2.8251E+02	RESIDUAL SS	= 4.7864E+01
MIN VIF	= 3.8496E+01	RBARSQ	= 9.7356E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.5511026E+00	5.5468215E-01	6.0651195E-01
2	5.1016758E-01	5.2386907E-01	5.2770563E-01
3	1.0190940E-01	5.6958574E-01	4.3389698E-02
4	-1.4406103E-01	5.0275483E-01	-1.6028742E-01
U	6.2405369E+01		.0

RIDGED (BIASED) SOLUTIONS

K	= 5.0000E-03	DETERMINANT	= 4.4962E-03
MAX VIF	= 1.7405E+01	RESIDUAL SS	= 4.8287E+01
MIN VIF	= 4.7340E+00	RBARSQ	= 9.7333E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.3551314E+00	6.8210719E-02	5.2988329E-01
2	3.3002834E-01	3.2933221E-02	3.4137373E-01
3	-9.3299737E-02	6.1180586E-02	-3.9723983E-02
4	-3.2009801E-01	3.0974122E-02	-3.5615241E-01
U	8.0120585E+01		.0

K	= 1.0000E-02	DETERMINANT	= 8.1399E-03
MAX VIF	= 5.9488E+00	RESIDUAL SS	= 4.8527E+01
MIN VIF	= 3.1275E+00	RBARSQ	= 9.7320E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.3152096E+00	4.5587520E-02	5.1427310E-01
2	3.0611536E-01	1.1685307E-02	3.1663870E-01
3	-1.2901810E-01	3.8007597E-02	-5.4931695E-02
4	-3.4293876E-01	1.0586404E-02	-3.8156584E-01
U	8.2675564E+01		.0

DATA FROM HALD GIVEN BY DRAPER AND SMITH (1966) P.39?

RIDGED (BIASED) SOLUTIONS

K = 2.0000E-02 DETERMINANT = 1.6084E-02

MAX VIF = 2.4564E+00 RESIDUAL SS = 4.9073E+01

MIN VIF = 2.0152E+00 RBARSQ = 9.7290E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.2722658E+00	3.5392970E-02	4.9748122E-01
2	2.9318939E-01	4.3406424E-03	3.0326837E-01
3	-1.6304659E-01	2.8226964E-02	-6.9419919E-02
4	-3.5429372E-01	3.5861906E-03	-3.9419977E-01
U	8.4359564E+01		.0

K = 3.0000E-02 DETERMINANT = 2.4926E-02

MAX VIF = 2.1661E+00 RESIDUAL SS = 4.9812E+01

MIN VIF = 1.1506E+00 RBARSQ = 9.7249E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.2413443E+00	3.1211060E-02	4.8539028E-01
2	2.8972330E-01	2.6871716E-03	2.9968313E-01
3	-1.8517787E-01	2.4617229E-02	-7.8842696E-02
4	-3.5624450E-01	2.0476469E-03	-3.9637028E-01
U	8.5076184E+01		.0

RIDGED (BIASED) SOLUTIONS

K = 4.0000E-02 DETERMINANT = 3.4689E-02

MAX VIF = 1.9668E+00 RESIDUAL SS = 5.0734E+01

MIN VIF = 8.2047E-01 RBARSQ = 9.7198E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.2151068E+00	2.8338752E-02	4.7513091E-01
2	2.8867933E-01	2.0300200E-03	2.9860327E-01
3	-2.0287118E-01	2.2278740E-02	-8.6375926E-02
4	-3.5571091E-01	1.4601020E-03	-3.9577659E-01
U	8.5514455E+01		.0

K = 5.0000E-02 DETERMINANT = 4.5398E-02

MAX VIF = 1.8077E+00 RESIDUAL SS = 5.1815E+01

MIN VIF = 6.5617E-01 RBARSQ = 9.7138E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.1917230E+00	2.6047364E-02	4.6598736E-01
2	2.8850399E-01	1.6856488E-03	2.9842190E-01
3	-2.1796008E-01	2.0465513E-02	-9.2800290E-02
4	-3.5423280E-01	1.1677159E-03	-3.9413199E-01
U	8.5830620E+01		.0

DATA FROM HALD GIVEN BY DRAPER AND SMITH (1966) P.395

RIDGED (BIASED) SOLUTIONS

K = 6.0000E-02 DETERMINANT = 5.7078E-02

MAX VIF = 1.6736E+00 RESIDUAL SS = 5.3034E+01

MIN VIF = 5.6025E-01 RBARSQ = 9.7071E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.1704313E+00	2.4114651E-02	4.5766190E-01
2	2.8867296E-01	1.4726538E-03	2.9859669E-01
3	-2.3119084E-01	1.8958139E-02	-9.8433514E-02
4	-3.5233834E-01	9.9701344E-04	-3.9202415E-01
U	8.6080234E+01		.0

K = 7.0000E-02 DETERMINANT = 6.9755E-02

MAX VIF = 1.5575E+00 RESIDUAL SS = 5.4371E+01

MIN VIF = 4.9788E-01 RBARSQ = 9.6997E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.1508131E+00	2.2441313E-02	4.4999079E-01
2	2.8896914E-01	1.3258173E-03	2.9890304E-01
3	-2.4296415E-01	1.7663335E-02	-1.0344620E-01
4	-3.5025225E-01	8.8601568E-04	-3.8970309E-01
U	8.6288334E+01		.0

RIDGED (BIASED) SOLUTIONS

K = 8.0000E-02 DETERMINANT = 8.3454E-02

MAX VIF = 1.4554E+00 RESIDUAL SS = 5.5813E+01

MIN VIF = 4.5406E-01 RBARSQ = 9.6917E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.1325960E+00	2.0970851E-02	4.4286754E-01
2	2.8929203E-01	1.2167696E-03	2.9923703E-01
3	-2.5353510E-01	1.6530643E-02	-1.0794697E-01
4	-3.4808308E-01	8.0802842E-04	-3.8728960E-01
U	8.6468050E+01		.0

K = 9.0000E-02 DETERMINANT = 9.8201E-02

MAX VIF = 1.3649E+00 RESIDUAL SS = 5.7348E+01

MIN VIF = 4.2143E-01 RBARSQ = 9.6833E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.1155836E+00	1.9665904E-02	4.3621533E-01
2	2.8959250E-01	1.1314068E-03	2.9954783E-01
3	-2.6308444E-01	1.5528094E-02	-1.1201277E-01
4	-3.4588760E-01	7.4996617E-04	-3.8484682E-01
U	8.6627044E+01		.0

DATA FROM HALD GIVEN BY DRAPER AND SMITH (1966) P.395

RIDGED (BIASED) SOLUTIONS

K = 1.0000E-01   DETERMINANT = 1.1402E-01
MAX VIF = 1.2839E+00   RESIDUAL SS = 5.8967E+01
MIN VIF = 3.9603E-01   RBARSQ = 9.6743E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.0996246E+00	1.8499396E-02	4.2997506E-01
2	2.8984628E-01	1.0619790E-03	2.9981034E-01
3	-2.7174930E-01	1.4633257E-02	-1.1570198E-01
4	-3.4369689E-01	7.0476959E-04	-3.8240937E-01
U	8.6770159E+01		.0

K = 1.2000E-01   DETERMINANT = 1.4899E-01
MAX VIF = 1.1453E+00   RESIDUAL SS = 6.2428E+01
MIN VIF = 3.5859E-01   RBARSQ = 9.6552E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.0704034E+00	1.6502951E-02	4.1854899E-01
2	2.9017524E-01	9.5419632E-04	3.0015061E-01
3	-2.8684109E-01	1.3103227E-02	-1.2212758E-01
4	-3.3939198E-01	6.3813896E-04	-3.7761957E-01
U	8.7020825E+01		.0

RIDGED (BIASED) SOLUTIONS

K = 1.5000E-01   DETERMINANT = 2.1017E-01
MAX VIF = 9.8164E-01   RESIDUAL SS = 6.8100E+01
MIN VIF = 3.2089E-01   RBARSQ = 9.6239E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.0320312E+00	1.4144186E-02	4.0354469E-01
2	2.9020376E-01	8.3893726E-04	3.0018010E-01
3	-3.0501923E-01	1.1295281E-02	-1.2986724E-01
4	-3.3321538E-01	5.7104445E-04	-3.7074726E-01
U	8.7334413E+01		.0

K = 2.0000E-01   DETERMINANT = 3.3721E-01
MAX VIF = 7.8682E-01   RESIDUAL SS = 7.8622E+01
MIN VIF = 2.8085E-01   RBARSQ = 9.5657E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	9.7878945E-01	1.1337044E-02	3.8272612E-01
2	2.8918811E-01	7.1099581E-04	2.9912954E-01
3	-3.2678834E-01	9.1385108E-03	-1.3913581E-01
4	-3.2371903E-01	4.9978694E-04	-3.6018129E-01
U	8.7751900E+01		.0

```
DATA FROM HALD GIVEN BY DRAPER AND SMITH (1966) P.375

WEIGHTED REGRESSION THRU THE MEAN    USING     DATA FROM PREVIOUS PROBLEM

NO. OF VARIABLES READ IN =  5 ON CARDS


VARIABLES IN REGRESSION

                1    1    2    2    3    3    4    4    5
           5    0    5    0    5    0    5    0    5    0
====XX     Y

NO. OF K VALUES READ IN =  3



LEAST-SQUARES SOLUTION


MSE     =  5.7904E+00          DETERMINANT =  9.4775E-01

MAX VIF =  1.0551E+00          RESIDUAL SS =  5.7904E+01

MIN VIF =  1.0551E+00          RBARSQ      =  9.7441E-01
```

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.4683057E+00	1.4713914E-02	5.7413672E-01
2	6.6225049E-01	2.1026555E-03	6.8501670E-01
U	5.2577349E+01		.0

DATA FROM HALD GIVEN BY DRAPER AND SMITH (1966) P.375

RIDGED (BIASED) SOLUTIONS

K = 1.1000E-01   DETERMINANT = 1.1799E+00

MAX VIF = 8.3931E-01   RESIDUAL SS = 7.5967E+01

MIN VIF = 8.3931E-01   RBARSQ = 9.6643E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.3536883E+00	1.1704247E-02	5.2931903E-01
2	6.0554453E-01	1.6725665E-03	6.2636136E-01
U	5.6163181E+01		.0

K = 1.3000E-01   DETERMINANT = 1.2247E+00

MAX VIF = 8.0750E-01   RESIDUAL SS = 8.2390E+01

MIN VIF = 8.0750E-01   RBARSQ = 9.6359E-01

REGRESSOR VARIABLE SUBSCRIPT	NORMAL REGRESSION COEFFICIENTS	VARIANCE OF NORMAL COEFFICIENTS	STANDARDIZED REGRESSION COEFFICIENTS
1	1.3346868E+00	1.1260710E-02	5.2188905E-01
2	5.9627993E-01	1.6091839E-03	6.1677827E-01
U	5.6751088E+01		.0

Mitchell, Brian R., and David W. Hann.
1979. BREX: A computer program for applying ridge regression techniques to multiple linear regression. USDA For. Serv. Gen. Tech. Rep. INT-51, 25 p. Intermt. For. and Range Exp. Stn., Ogden, Utah 84401.

An algorithm for ridge regression (BREX) is presented which can be used in conjunction with Grosenbaugh's REX (1967), a linear regression program with combinatorial screening. The algorithm uses either REX punched matrix input, or raw data with suitable transformations, to estimate ridged (biased) regression coefficients.

KEYWORDS: ridge regression, biased estimation, multicollinearity, linear regression, least-squares estimation, computer program

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Headquarters for the Intermountain Forest and Range Experiment Station are in Ogden, Utah. Field programs and research work units are maintained in:

- Billings, Montana
- Boise, Idaho
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- Logan, Utah (in cooperation with Utah State University)
- Missoula, Montana (in cooperation with University of Montana)
- Moscow, Idaho (in cooperation with the University of Idaho)
- Provo, Utah (in cooperation with Brigham Young University)
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